1 Introduction

The relationship between “rms• asymmetries and collusive behavior has been at the center of attention for antitrust practitioners as well as strategy theorists. In this paper, we investigate cartel stability in quality di erentiated industries.

Many detected cartels refer to industries that exhibit market-share asymmetries[1], which are often due to vertical di erentiation in product performance, brand image, or reputation. Quality is an important parameter which plays an important role in market decisions.[2] In digital and technology markets where production costs are falling[3] and big data analytics are extensively used, cost asymmetry becomes less relevant, and quality di erentiation emerges as one of the important parameters for de“ning market strategies.

Two important questions relate to how the degree of quality di erentiation a ects the stability of cartels, and which “rm - the innovative leader or a technological laggard - is more likely to abandon the collusive agreement.

The scarce literature on the topic (Häckner, 1994; Symeonidis, 1999; Bos and Marini, 2019, Given the trade-o between static joint pro“t maximization and dynamic stability when “rms are asymmetric, a more appropriate method to study collusion is the Nash bargaining approach (Nash, 1950). Nash bargaining allows us to focus on the set of implementable subgame perfect Nash equilibrium collusive strategies. When the set of subgame perfect equilibria is large, as it is often the case in repeated game settings, it is natural to consider the “rms to engage in bargaining over the set of potential outcomes (Harrington, 1991).

We adopt an in“nite time horizon and we consider two “rms with di erent quality levels (which we call the leader with high quality and the follower with low quality) that either compete in prices or collude, without any inter-“rm payments. We analytically derive the Nash bargaining solution that jointly determines the level and the division of collusive pro“ts.[4] Nash bargaining allows for the endogenous derivation of implementable collusive prices weighting both static pro“ts and dynamic incentives for collusion, without having to rely on additional assumptions that may be di cult to justify.[5]

Assessing the stability of the derived collusive agreement requires to specify the optimal pun-

We also show that it is the follower (leader) who has higher incentives to deviate from the collusive agreement if the quality di erence between the two “rms• goods is relatively low (high).

As the quality di erence increases, the collusive price of the leader and the follower diverges. As more consumers will prefer the high-quality product, the collusive agreement adjusts the two prices so that the follower will keep its consumer base and will be collusive ì •Ÿ agreement than a less efficient firm, as it has been found by the literature.

The Nash bargaining solution has already been implemented in the literature that deals with cost asymmetry (Schmalensee, 1987, Harrington, 1991, Miklós-Thal, 2011). The main conclusion in these papers is that cost asymmetry hinders collusion and that it is the least efficient firm that has more incentives to deviate from the collusive equilibrium. But, quality differentiation, unlike cost asymmetry, directly affects consumer preferences, which we model explicitly. Under cost asymmetry, consumers have to choose among identical products and their product choices are and Section 7 concludes.

2 The Model

There are two firms, the leader (L) and the follower (F), interacting repeatedly in the same market over an infinite, discrete-time horizon. The stage game models a vertically differentiated industry setting in the tradition of Shaked and Sutton (1982). Each firm supplies a single product whose quality is given by q_i, $i = L, F$, with $q_L \quad q_F$. Our primary interest is in cases with $q_L > q_F$, while we briefly study the symmetric case with $q_L = q_F \quad q_0$. We assume quality levels to be exogenously given. Firms simultaneously choose prices p_L and p_F to maximize the discounted sum of period profits $\ _L$ and $\ _F$. The marginal costs of production for all products are normalized to zero[9], and firms have a common discount factor $\delta \quad (0, 1)$. We denote the degree of differentiation by $k \quad q_L/q_F$.

There is a continuum of heterogeneous consumers who differ in their valuations θ for product quality, where θ

the punishment phase, firms earn their competitive profits.

The sustainability of collusion requires

$$\Pi_i^c \geq (1-\delta)\,\Pi_i^d + \delta\,\Pi_i^*, \tag{6}$$

for each $i = L, F$, where the superscript (*) denotes the punishment phase, (c) denotes collusion, and (d) denotes deviation. Condition (6) implies that firm i does not deviate as long as

$$\delta \geq \delta_i = \frac{\Pi_i^d - \Pi_i^c}{\Pi_i^d - \Pi_i^*}, \tag{7}$$

where δ_i is a firm-specific threshold discount factor measuring the incentives of firm $i = L, F$ to deviate from the collusive agreement. Hence, the stability of collusion is determined by the discount factor $\delta \geq \max\{\delta_F, \delta_L\}$.

Proposition 1 shows that in our framework the grim trigger punishment (Friedman, 1971) is the optimal punishment mechanism in the sense of Abreu (1986, 1988), and therefore dominates any form of stick-and-carrot punishment.

Proposition 1. The optimal punishment mechanism is the grim trigger punishment. Following the deviation, firms revert to the static Nash equilibrium for all the subsequent periods.

Proof. We define as the optimal mechanism, the one that i) minimizes the expected payoff of the deviator; 2) it is credible such that the payoff of the non-deviator in the punishment phase is sufficiently high to implement that punishment. It suffices to show that there cannot be a more severe punishment for the deviator which is at the same time credible for the non-deviator. Let us assume that the leader deviates. The payoff of the leader and the follower under the grim trigger strategies, in the punishment phase, will be: $\frac{\Pi_L^*}{1-\delta}$ and $\frac{\Pi_F^*}{1-\delta}$, respectively.

Following Abreu (1986) a natural candidate mechanism will be the one which punishes harshly the deviator for the first τ periods of the punishment phase (the stick). Given expression (3), the most harsh punishment for the leader will be the follower to set $p_F = 0$ for the first τ periods. Then, for each period $t \leq \tau$, the leader gets payoff $\frac{q_F(k-1)}{4}$ which is smaller than Π_L^*. For $t > \tau$, let the follower charge price $p_F^o > 0$. This mechanism can be optimal only if the following two conditions are satis“ed:

$$\frac{q_F\,(k - 1)}{4}(1 - \quad) + \quad .9-$$

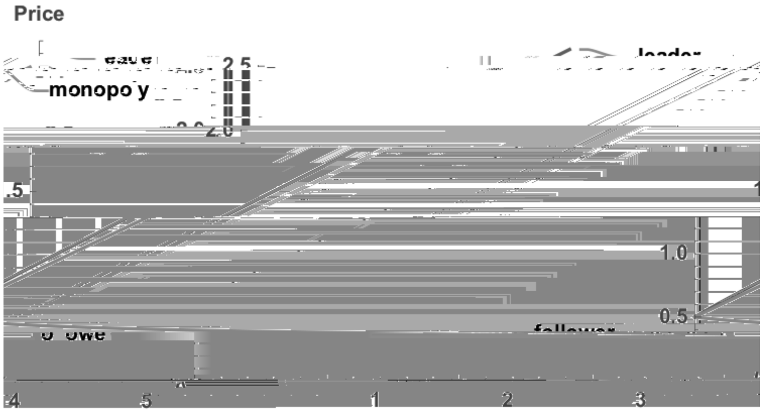


Figure 1: Collusive prices of leader and follower and monopoly price of leader as functions of the quality differentiation (k).

leads to the following Nash bargaining sharing rule:

$$\max_{p_L^c, p_F^c} \{(\pi_L^c - \bar{\pi}_L)(\pi_F^c - \bar{\pi}_F)\} \quad \text{s.t.} \quad \pi_L^c > \bar{\pi}_L, \quad \pi_F^c > \bar{\pi}_F, \tag{10}$$

where

$$\pi_L^c = p_L^c \left(1 - \frac{p_L^c - p_F^c}{q_L - q_F}\right), \qquad \pi_F^c = p_F^c \left(\frac{p_L^c - p_F^c}{q_L - q_F} - \frac{p_F^c}{q_F}\right), \tag{11}$$

and p_L^c and p_F^c denote equilibrium collusive prices.

The bargaining problem in (10) leads to analytical solutions for collusive prices p_L^c and p_F^c.[10] The marginal consumers θ^c, θ_L^c and θ_F^c that determine demand functions are calculated using collusive prices, and satisfy $\theta^c > \theta_L^c > \theta_F^c$. Figure 1 depicts $\frac{p_L^c}{q_F}$ and $\frac{p_F^c}{q_F}$ as well as $\frac{p_L^m}{q_F}$, where p_L^m is the monopoly price of the leader.[11]

The leader's collusive price is increasing in quality differentiation k while the respective price for the follower is decreasing. Thus, as the quality advantage increases, so does the equilibrium price

Expusss.81[(di41s8n)a01 36 (o61 0 0 299)54o 36 (o (11)ngth/T8_57.3 (s4es8n)to 36 3)-41b /T1_40.264 4(o6 interesting feature of the collusive equilibrium is that the leader charges a price that exceeds its monopoly price. The leader is willing to forgo a part of the monopoly profit by charging a higher price so that the follower has sufficient incentives to participate in the collusive equilibrium without deviating. The difference between the leader's collusive and monopoly price is increasing in k.

4.3 Deviation strategies

The optimal deviation strategy for each firm $i = L, F$ is to select the price that maximizes its profits given the rival firm's collusive price p_j^c, where $j = L, F$ and $j \neq i$. The deviator's best response to the other firm playing its collusive equilibrium strategy could potentially be an interior price choice - coming from the first-order conditions of its profit maximization problem - or a price that could force the competitor to have zero demand. This leads to:

$$p_L^d = \begin{cases} \frac{p_F^c}{2} + \frac{q_F(k-1)}{2} & \text{if} \quad p_F^c \leq \frac{q_F(k-1)}{2k-1}, \\ k p_F^c & \text{if} \quad \frac{q_F(k-1)}{2k-1} < p_F^c, \end{cases} \tag{12}$$

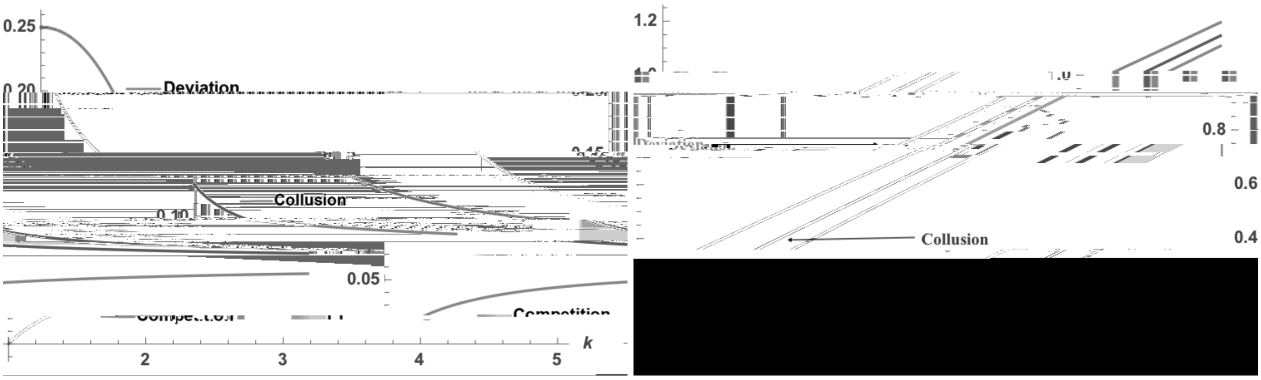
for the leader, and

$$p_F^d = \begin{cases} \frac{p_L^c}{2k} & \text{if} \quad p_L^c \leq q_F \frac{2k(k-1)}{2k-1}, \\ p_L^c - q_F(k-1) & \text{if} \quad q_F \frac{2k(k-1)}{2k-1} < p_L^c, \end{cases} \tag{13}$$

for the follower.[12]

When the deviation does not violate the constraint $\theta_F \leq \theta \leq 1$, it is best for a firm to deviate according to the best response functions in (3), by maximizing own profits holding rival's price at its collusive level. These are stated by the first interval in the deviation functions above. However, price levels may be such that the deviating firm can push its rival to have zero demand in the deviation period. For the follower, this occurs if the best response function leads θ^d, the consumer that is indifferent between the two products, to be equal to θ_F^c, essentially leaving the leader with zero demand. In this price range, the follower undertakes a form of limit pricing to keep the leader's demand at zero and serve all consumers with $\theta \in [\theta_F, 1]$ in the deviation period. A similar, but slightly different strategy exists for the leader, whose limit pricing deviation leads to the binding

[12] Note that equilibrium collusive prices satisfy $p_F^c \geq \frac{q_F}{2}$ and $p_L^c \geq q_F \frac{2k-1}{2k-1}$, $k > 1$. We present the deviation strategies that may arise given the collusive equilibrium strategies. If we also include deviations off equilibrium paths in the analysis, there is a third deviation strategy for the leader (when follower's collusive price is greater than $\frac{q_F}{2}$) and the follower (when leader's collusive price greater than $\frac{2k-1}{2k-1}$) for which the deviator charges its monopoly price.



(a) (b)

Figure 2: Collusive, competitive and deviation profits for (a) the follower and (b) the leader as a function of k.

constraint $\theta = \theta_F$, which effectively keeps F out of the market in the deviation period.

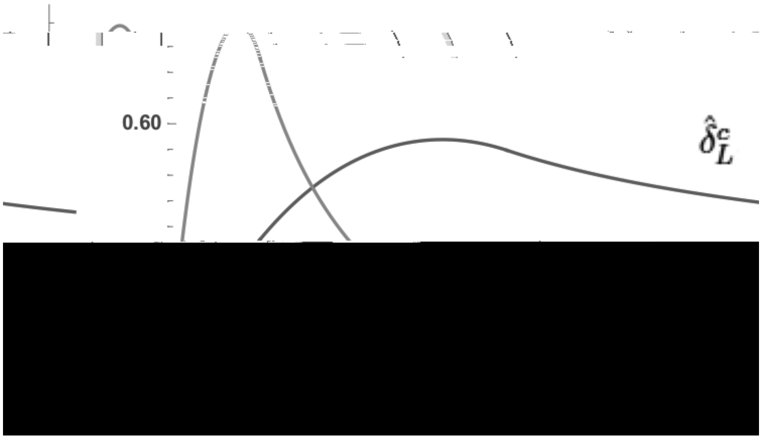


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Figure 3: The critical discount factors of the leader (blue) and the follower (orange) as functions of quality di erentiation (k).

an inverted-U pattern with k. The peak occurs at a lower value of k for the follower. For each “rm, as k increases, while the one-period deviation becomes a relatively less attractive option, the punishment (competitive) payo becomes relatively more attractive. For low-quality di erences, it is the latter e ect that dominates and collusion becomes less stable with k. For high-quality di erences, it is the former e ect that dominates and hence collusion becomes more stable.

Indeed, the di erence between the one-period deviation pro“t and static collusive pro“t declines at a lower (higher) rate than the di erence between static pro“ts under collusion and competition for both “rms when quality di erentiation is low (high).

Furthermore, the “rm that determines cartel stability depends on the degree of quality di erentiation. More precisely, for lower values precisely ð qualit6y qualit73 957r stability. So, the relationship between cartel stability and k is non-monotonic as well:

Proposition 3. There exist cutoffs $\bar{k} = 1.426, k = 1.829, k = 2.65$, such that the cartel becomes (a) more stable with increased quality differentiation when $\bar{k} < k < k$ or $k > k$, (b) less stable with vertical differentiation when $k < \bar{k}$ or $k < k < k$.

These results deviate from the literature in vertically differentiated industries, according to which i) there is a monotonic relationship between the quality asymmetry and collusion (i.e., Häckner, 1994; Symeonidis, 1999; Ecchia and Lambertini, 1997), and ii) a single firm has uniformly higher incentives to abandon the cartel: either the high-quality firm (Häckner, 1994) or the technological laggard (Symeonidis, 1999; Bos and Marini, 2019). Adopting a Nash bargaining approach that allows to determine endogenously the collusive equilibrium instead of computing this equilibrium in an ad hoc way provides new insights on firms' equilibrium strategies that have direct implications for incentives to collude.

5 Different marginal costs

In this section, we incorporate non-zero marginal costs of production to our baseline model presented in the previous sections. We characterize the collusive equilibrium and its stability when the two firms have different marginal costs, denoted c_L (leader) and c_F (follower). It is natural to consider marginal costs of production to increase with product quality, hence to assume $c_F \leq c_L$.[13]

In the presence of non-zero marginal costs, the following two constraints need to be satisfied for

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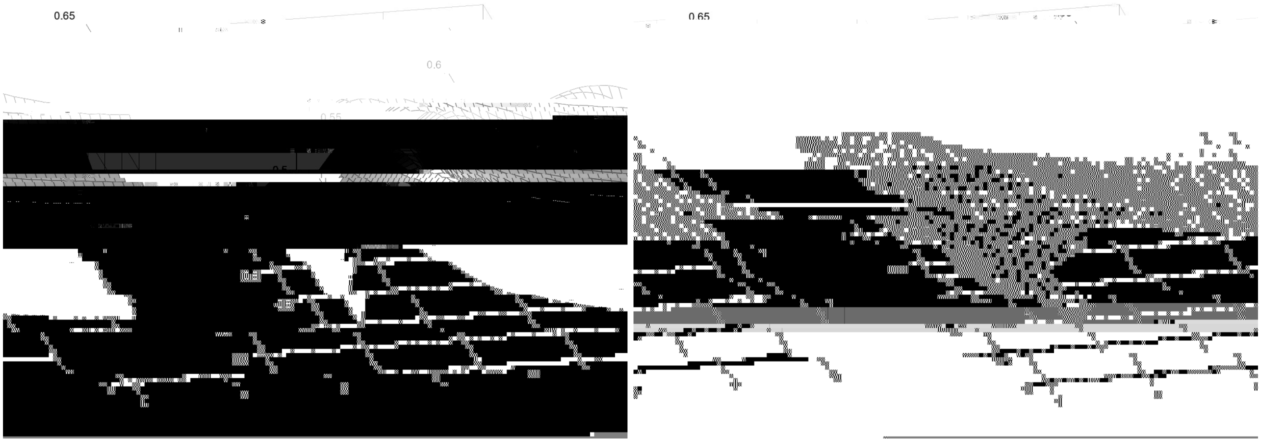

(a) (b)

Figure 4: Threshold discount factors of (a)

the leader (follower), hence makes the deterrence e ect stronger (weaker) for this “rm. Hence, the leader has weaker, while the follower has stronger incentives to deviate compared to our baseline model. This e ect is more dominant for larger k which leads to equilibrium collusive profits

$$\Pi_L^{sp} = \frac{8q_L^2 - 5q_L q_F}{8(4q_L - q_F)}, \qquad \Pi_F^{sp} = \frac{3q_L q_F}{8(4q_L - q_F)}. \tag{17}$$

Note that the collusive participation constraints are satisfied for all q_L and q_F with $q_L > q_F > 0$. To implement the strategy, the leader makes all sales and pays an amount equal to Π_F^{sp} in (17) to the follower in each period.

It is easy to see that the leader's optimal deviation from the collusive agreement is to refuse to make the side payment to the follower, which gives the deviation profit $\Pi_L^{d,sp} = \Pi^{sp} = q_L/4$. The optimal deviation strategy of the follower is derived in an analogous way to the previous section. This leads to the two-part deviation profits

$$\Pi_F^{d,sp} = \begin{cases} \frac{q_L(2q_F - q_L)}{4q_F} & \text{if } 1 < k < \frac{3}{2}, \\ \frac{q_L q_F}{16(q_L - q_F)} & \text{if } \frac{3}{2} \leq k. \end{cases} \tag{18}$$

Note that $\Pi_L^{d,sp} > \Pi_L^{sp}$ for all $q_L > q_F > 0$. However, $\Pi_F^{d,sp} > \Pi_F^{sp}$ only if $k < \frac{5}{2}$. For higher differentiation with $k \geq \frac{5}{2}$, the follower never deviates from the collusive agreement.

The critical discount factors for both firms can then be computed using (7) as

$$\delta_L^{sp} = \frac{3}{4}\left(1 - \frac{3}{1+8k}\right) \tag{19}$$

and

$$\delta_F^{sp} = \begin{cases} 1 - \frac{4k+5}{12 \quad 42k+80k^2 \quad 32k} \end{cases}$$

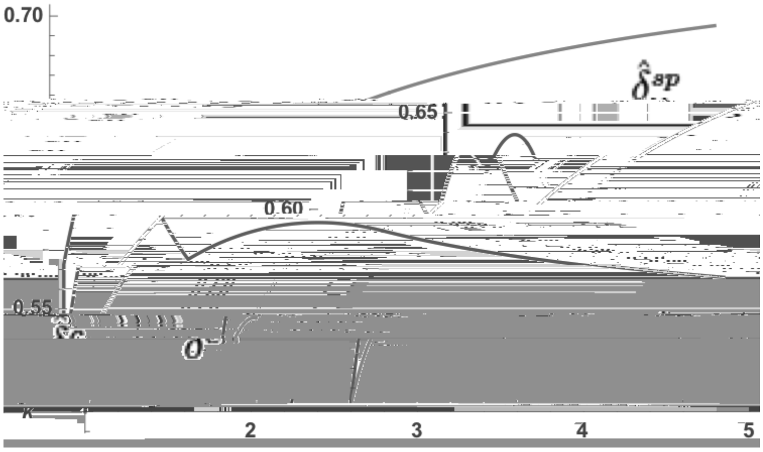

Figure 5: Critical discount factors for the stability of collusion with and without side payments as a function of quality di erentiation (k).

Comparing the stability of collusion, δ^c of our baseline model above with the side payments case, δ^{sp} (Figure 5) we seethat:

Proposition 4. There is a cuto value for quality asymmetry, $k = 1.708$, abovewhich collusion is more stable in the absenceof side payments.

This indicates that side payments can lead to the destabilization of the collusive ,¡HY Às •that$tupÝÁ†# Þ«¬ "

agreement for low (high) degrees of di erentiation between competitors. We also “nd that side payments can render collusion more stable only if product qualities in the industry are su ciently close to one another.

Understanding the incentives to collude is important for organizing deterrence mechanisms that promote competition. In this respect, our model predictions shed light on the incentives of market leaders and followers to collude, in cases the quality of products and services is an important strategic variable (as in digital ecosystems and technology markets). In many instances, deterrence of collusive agreements relies on identifying potential whistle-blowers within the “rms that only have weak incentives to collude.

Our approach and results have important implications for future research. The literature on the relationship between collusion and innovation largely deals with cost-reducing innovation. Our analysis paves the way for investigating the relationship between collusion and innovation when innovation improves a product in technological performance or in use-value. Extending our model to study the relationship between R&D competition and collusion on a learning curve (e.g., by adding a quality investment step in each “rm•s decision problem per period) is part of our current research e orts.

The computational di culties introduced by the general market setting restricted our e orts to the case of a duopoly. The generalization of our model to an oligopoly with an arbitrary number of “rms is also part of our ongoing research.

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